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A BROADBAND BEAMFORMER USING CONTROLLABLE CONSTRAINTS AND MINIMUM VARIANCE

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ABSTRACT

The minimum variance distortionless response (MVDR) and the linearly constrained minimum variance (LCMV) beamformers are two optimal approaches in the sense of noise reduction. The LCMV beamformer can also reject interferers using linear constraints at the expense of reducing the degree of freedom in a limited number of microphones. However, it may magnify noise that causes a lower output signal-to-noise ratio (SNR) than the MVDR beamformer. Contrarily, the MVDR beamformer suffers from interference in output. In this paper, we propose a controllable LCMV (C-LCMV) beamformer based on the principles of both the MVDR and LCMV beamformers. The C-LCMV approach can control a compromise between noise reduction and interference rejection. Simulation results show that the C-LCMV beamformer outperforms the MVDR beamformer in interference rejection, and the LCMV beamformer in background noise reduction.

Index Terms— Microphone arrays, frequency-domain beamforming, MVDR, LCMV, controllable beamformer.

1. INTRODUCTION

Multiple acoustic sources are usually present in real situations. For speech processing applications such as teleconferencing and hearing aids, noise reduction techniques are developed to achieve a high quality and preserve the intelligibility of the desired signal. In single-channel signal enhancement methods, both the desired signal and noise are filtered at the same time [1]. While the speech quality is increased in the Wiener filter, which is an example of a known noise-reduction filter [1, 2], speech distortion will be increased in the presence of interference. Exploiting spatial separation is another solution to separate multiple signals and enhance the desired signal using multiple microphones, which is called a microphone array.

Beamforming is one of the techniques for microphone arrays to estimate the signal arriving from a desired direction-of-arrival (DOA), and separate different signal sources [3]. The basic principle is that the received signals through multiple microphones are synchronized by delays depending on

the desired DOA using complex weighted filters and summed, e.g., as in the delay-and-sum (DS) beamforming [4]. Besides the spatial separation, signal enhancement is another issue in the filter design, where the adaptive filters are designed to minimize the noise and interference using the statistics of the received signals. An adaptive multichannel filter can provide a trade-off between noise reduction and signal distortion [5], e.g., the multichannel Wiener filter [6], and the maximum SNR filter [4]. Some well-known examples of beamformer designs are the least-squares, multiple sidelobe canceler (MSC) [7], generalized sidelobe canceler (GSC) [8, 9], superdirective [10], minimum variance distortionless response (MVDR) [11], and linearly constrained minimum variance (LCMV) [12] beamformers. For more details about various beamformer designs, we refer the reader to [4] and [13].

In this paper, we propose a new beamformer based on the principles of the MVDR and LCMV beamformers using the spectral decomposition [14–16]. Both are designed to minimize the output power subject to a unit output gain at the desired DOA, and through exploiting the decomposition of interfering signals, we can have multiple constraints to reject the interference in the LCMV. Although the number of constraints degrades degree of freedom (DOF) of a beamformer which is as many as the number of microphones [17]. Though there is a trade-off between noise and interference reduction, the LCMV beamformer may magnify the background noise [18] with having high sidelobes [19]. Therefore, we explore a new flexible beamformer based on the paradigm of minimum variance in order to control the output signal-to-interference-plus-noise ratio (SINR) and signal-to-interference ratio (SIR). That is, we propose the controllable LCMV (C-LCMV) beamformer with a variable number of constraints.

The rest of this paper is organized as follows. In Section 2, we model the composition of multiple signal sources in vector notation and design the MVDR and LCMV beamformers accordingly. In Section 3, we propose the C-LCMV beamformer, and then explore the properties of this method in simulations in Section 4. The work is concluded in Section 5.

2. PROBLEM FORMULATION

2.1. Signal model

We consider a microphone array, consisting of M omnidirectional microphones, receives broadband signals from N acoustic sources besides a background noise, where $N \leq M$. In general, we model the received signals at the frequency index f in a vector notation as $\mathbf{y}(f) = [Y_1(f) Y_2(f) \cdots Y_M(f)]^T$, where $Y_m(f)$ is the m th microphone narrowband signal and the superscript T is the transpose operator. We write the vector $\mathbf{y}(f)$ as a function of the (known) steering vectors $\mathbf{d}_n(f)$ and signal sources $X_n(f)$ for $n = 1, \dots, N$ [4, 16] like

$$\begin{aligned} \mathbf{y}(f) &= \mathbf{d}_1(f)X_1(f) + \sum_{n=2}^N \mathbf{d}_n(f)X_n(f) + \mathbf{v}(f) \\ &= \mathbf{D}(f)\mathbf{x}(f) + \mathbf{v}(f), \end{aligned} \quad (1)$$

where $\mathbf{v}(f) = [V_1(f) V_2(f) \cdots V_M(f)]^T$ is the additive background noise, $\mathbf{x}(f) = [X_1(f) X_2(f) \cdots X_N(f)]^T$ is the collection of signal sources, and we define $\mathbf{D}(f)$ as the $M \times N$ matrix containing all steering vectors relating to the N signal sources, i.e.,

$$\mathbf{D}(f) = [\mathbf{d}_1(f) \mathbf{d}_2(f) \cdots \mathbf{d}_N(f)]. \quad (2)$$

We assume that $X_n(f)$ and $V_m(f)$ are uncorrelated and zero mean. Furthermore, we consider $X_1(f)$ as the desired signal that we wish to extract from the observations, while $X_n(f)$ for $n = 2, 3, \dots, N$ are interferers.

The correlation matrix of $\mathbf{y}(f)$ is defined as $\Phi_{\mathbf{y}}(f) = E[\mathbf{y}(f)\mathbf{y}^H(f)]$, where $E[\cdot]$ denotes mathematical expectation, and the superscript H is the transpose-conjugate operator. If we assume all signal sources and noise are uncorrelated, we can write the correlation matrix as

$$\begin{aligned} \Phi_{\mathbf{y}}(f) &= \mathbf{D}(f) \Phi_{\mathbf{x}}(f) \mathbf{D}^H(f) + \Phi_{\mathbf{v}}(f) \\ &= \mathbf{d}_1(f) \phi_{X_1}(f) \mathbf{d}_1^H(f) + \Phi_{\text{in}}(f) + \Phi_{\mathbf{v}}(f), \end{aligned} \quad (3)$$

where $\Phi_{\mathbf{x}}(f) = \text{diag}[\phi_{X_1}(f) \phi_{X_2}(f) \cdots \phi_{X_N}(f)]$ is a diagonal matrix of size $N \times N$ containing the variances of the sources at the frequency index f , i.e., $\phi_{X_n}(f) = E[|X_n(f)|^2]$, the correlation matrix of $\mathbf{v}(f)$ is $\Phi_{\mathbf{v}}(f) = E[\mathbf{v}(f)\mathbf{v}^H(f)]$, and $\Phi_{\text{in}}(f) = \sum_{n=2}^N \mathbf{d}_n(f) \phi_{X_n}(f) \mathbf{d}_n^H(f)$ is the interference correlation matrix. If the components of the steering vectors are only phase shifts, which is usually the case, then $\mathbf{d}_n^H(f)\mathbf{d}_n(f) = M$. As a result, we can deduce the narrowband input SIR and input SINR respectively like

$$\text{iSIR}(f) = \frac{\phi_{X_1}(f)}{\sum_{n=2}^N \phi_{X_n}(f)}, \quad (4)$$

$$\text{iSINR}(f) = \frac{M \phi_{X_1}(f)}{\text{tr}[\Phi_{\text{in}}(f) + \Phi_{\mathbf{v}}(f)]}, \quad (5)$$

where $\text{tr}[\cdot]$ denotes the trace of a square matrix.

We apply a complex-valued filter, or a beamformer as we refer to, $\mathbf{h}(f) = [H_1(f) H_2(f) \cdots H_M(f)]^T$ on the microphone outputs, that results $Z(f) = \mathbf{h}^H(f)\mathbf{y}(f)$ with the variance of

$$\begin{aligned} \phi_Z(f) &= \mathbf{h}^H(f) \mathbf{d}_1(f) \phi_{X_1}(f) \mathbf{d}_1^H(f) \mathbf{h}(f) + \\ &\quad \mathbf{h}^H(f) [\Phi_{\text{in}}(f) + \Phi_{\mathbf{v}}(f)] \mathbf{h}(f). \end{aligned} \quad (6)$$

With the distortionless constraint that $\mathbf{h}^H(f)\mathbf{d}_1(f) = 1$, we can write the narrowband output SIR and output SINR respectively like

$$\text{oSIR}[\mathbf{h}(f)] = \frac{\phi_{X_1}(f)}{\mathbf{h}^H(f) \Phi_{\text{in}}(f) \mathbf{h}(f)}, \quad (7)$$

$$\text{oSINR}[\mathbf{h}(f)] = \frac{\phi_{X_1}(f)}{\mathbf{h}^H(f) [\Phi_{\text{in}}(f) + \Phi_{\mathbf{v}}(f)] \mathbf{h}(f)}. \quad (8)$$

2.2. Minimum variance beamformers

A fixed beamformer is a signal independent filter with a specific beampattern, e.g., the DS beamforming has a unit gain at the desired DOA, i.e., $\mathbf{h}_{\text{DS}}(f) = \mathbf{d}_1(f)/M$. However the desired signal is obtained from the desired direction, the output signal suffers from interference-plus-noise except for the unlikely cases when the nulls of the DS beamformer are situated at the direction of interferers. Signal dependent beamformers are designed adaptively to minimize the variance of the output signal. The MVDR or the Capon method [11] minimizes the output interference-plus-noise variance of the beamformer [20], i.e.,

$$\begin{aligned} \min_{\mathbf{h}(f)} \quad & \mathbf{h}^H(f) [\Phi_{\text{in}}(f) + \Phi_{\mathbf{v}}(f)] \mathbf{h}(f) \\ \text{subject to} \quad & \mathbf{h}^H(f) \mathbf{d}_1(f) = 1, \end{aligned} \quad (9)$$

and the MVDR beamformer is given by [4]

$$\mathbf{h}_M(f) = \frac{[\Phi_{\text{in}}(f) + \Phi_{\mathbf{v}}(f)]^{-1} \mathbf{d}_1(f)}{\mathbf{d}_1^H(f) [\Phi_{\text{in}}(f) + \Phi_{\mathbf{v}}(f)]^{-1} \mathbf{d}_1(f)}. \quad (10)$$

In the MVDR filter design, interferers are assumed to be uncorrelated with the desired signal; otherwise the desired signal may be suppressed. Herein, we generalize the MVDR beamformer to derive the LCMV filter that nulls out $N - 1$ number of interferers and minimizes the noise variance, i.e.,

$$\begin{aligned} \min_{\mathbf{h}(f)} \quad & \mathbf{h}^H(f) \Phi_{\mathbf{v}}(f) \mathbf{h}(f) \\ \text{subject to} \quad & \mathbf{h}^H(f) \mathbf{D}(f) = \mathbf{i}_N^T, \end{aligned} \quad (11)$$

where \mathbf{i}_N is the first column of the $N \times N$ identity matrix, \mathbf{I}_N . The solution for the LCMV beamformer is

$$\mathbf{h}_L(f) = \Phi_{\mathbf{v}}^{-1}(f) \mathbf{D}(f) [\mathbf{D}^H(f) \Phi_{\mathbf{v}}^{-1}(f) \mathbf{D}(f)]^{-1} \mathbf{i}_N. \quad (12)$$

3. PROPOSED METHOD

The optimization procedures in the MVDR and the LCMV beamformers consist of the number of constraints and the residual (interference-plus-)noise. To design a beamformer which has properties between those beamformers, we now introduce a general expression for the signal model. We divide N signal sources into two sets of N_1 and N_2 sources as $\mathbf{x}(f) = [\mathbf{x}_{N_1}^T(f) \mathbf{x}_{N_2}^T(f)]^T$. Therefore, the received signals can be written like

$$\mathbf{y}(f) = \mathbf{D}_{N_1}(f) \mathbf{x}_{N_1}(f) + [\mathbf{D}_{N_2}(f) \mathbf{x}_{N_2}(f) + \mathbf{v}(f)], \quad (13)$$

where $\mathbf{D}_{N_1}(f)$ and $\mathbf{D}_{N_2}(f)$ are matrices containing the steering vectors of the related signal sets, i.e., $\mathbf{D}(f) = [\mathbf{D}_{N_1}(f) \mathbf{D}_{N_2}(f)]$. We can rewrite the correlation matrix of this decomposition as

$$\Phi_{\mathbf{y}}(f) = \mathbf{D}_{N_1}(f) \Phi_{\mathbf{x}_{N_1}}(f) \mathbf{D}_{N_1}^H(f) + \Phi_{\text{in},N_2}(f) + \Phi_{\mathbf{v}}(f), \quad (14)$$

where $\Phi_{\text{in},N_2}(f) = \mathbf{D}_{N_2}(f) \Phi_{\mathbf{x}_{N_2}}(f) \mathbf{D}_{N_2}^H(f)$, and $\Phi_{\mathbf{x}_{N_1}}(f)$ and $\Phi_{\mathbf{x}_{N_2}}(f)$ are the correlation matrices of the $\mathbf{x}_{N_1}^T(f)$ and $\mathbf{x}_{N_2}^T(f)$ signal sets, respectively.

We apply the signal decomposition model (13) to propose a beamformer which we call the controllable LCMV (C-LCMV) inspired from LCMV and MVDR beamformers. For the set of N_1 signal sources, containing the desired signal, the filter is constrained to null out the remaining $N_1 - 1$ interferers, and the remaining $N_2 = N - N_1$ signal sources are minimized together with the background noise, i.e.,

$$\begin{aligned} \min_{\mathbf{h}(f)} \quad & \mathbf{h}^H(f) [\Phi_{\text{in},N_2}(f) + \Phi_{\mathbf{v}}(f)] \mathbf{h}(f) \\ \text{subject to} \quad & \mathbf{h}^H(f) \mathbf{D}_{N_1}(f) = \mathbf{i}_{N_1}^T. \end{aligned} \quad (15)$$

The C-LCMV beamformer is designed using the method of Lagrange multipliers as

$$\begin{aligned} \mathbf{h}_C(f) = & [\Phi_{\text{in},N_2}(f) + \Phi_{\mathbf{v}}(f)]^{-1} \mathbf{D}_{N_1}(f) \\ & [\mathbf{D}_{N_1}^H(f) [\Phi_{\text{in},N_2}(f) + \Phi_{\mathbf{v}}(f)]^{-1} \mathbf{D}_{N_1}(f)]^{-1} \mathbf{i}_{N_1}. \end{aligned} \quad (16)$$

This optimal filter is controlled using a different number of constraints, i.e. $N_1 = 1, 2, \dots, N$. In particular cases, if $N_1 = 1$ or $N_1 = N$, the filter will be the MVDR beamformer or the LCMV beamformer, respectively. Therefore, the C-LCMV beamformer has the following properties:

$$\text{oSINR}[\mathbf{h}_L(f)] \leq \text{oSINR}[\mathbf{h}_C(f)] \leq \text{oSINR}[\mathbf{h}_M(f)], \quad (17)$$

$$\text{oSIR}[\mathbf{h}_M(f)] \leq \text{oSIR}[\mathbf{h}_C(f)] \leq \text{oSIR}[\mathbf{h}_L(f)]. \quad (18)$$

4. SIMULATION RESULTS

We investigate the performance of the C-LCMV beamformer comparing with the DS, MVDR, and LCMV beamformers

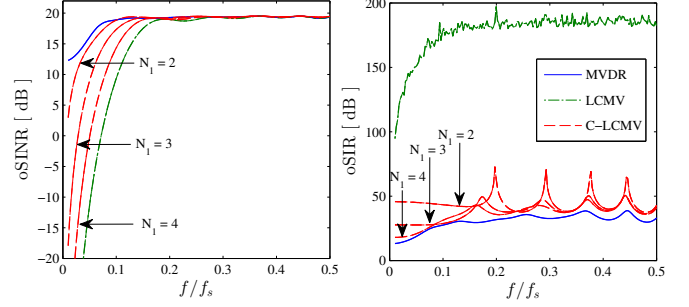


Fig. 1. Output SINR (left) and output SIR (right) of different beamformers versus frequency (input SINR = 8 dB, and input SIR = 13 dB).

in an anechoic environment. We use a uniform linear array (ULA) which the distance between microphones is $\delta = 0.04$ m, i.e., smaller than the half of the minimum wavelength to avoid spatial aliasing, and the wave propagation speed is assumed $c = 340$ m/s. By selecting the first microphone as the reference microphone, the steering vector $\mathbf{d}_n(f) = \mathbf{d}(f, \theta_n)$ can be written as a function of the DOA of the n th signal source, i.e., θ_n , as

$$\mathbf{d}_n(f) = [1 \ e^{-j2\pi f \tau_0 \cos \theta_n} \dots e^{-j2(M-1)\pi f \tau_0 \cos \theta_n}]^T, \quad (19)$$

where $j = \sqrt{-1}$, and $\tau_0 = \delta/c$ is the delay between two successive sensors at the zero angle.

In Figure 1, we plot narrowband oSINRs and oSIRs for various number of constraints N_1 , where $M = 9$, and $N = 5$ white Gaussian signal sources at $\theta_1 = \pi/6$, $\theta_2 = \pi/2$, $\theta_3 = 2\pi/3$, $\theta_4 = 5\pi/6$, and $\theta_5 = \pi$. This figure illustrates that the C-LCMV beamformer performs in the range between the MVDR and LCMV beamformers. In the next experiments, we use three speech signals and white Gaussian noise, which are located at θ_n (for $n = 1, 2, 3$, and 4), and synthesized according to the signal model (1). The desired speech signal is an utterance of “Then, the sun shine”, and interferers are utterances of “Why were you away?” and “Somebody decides to break it!”.

The speech signals were sampled at $f_s = 8.0$ kHz during 1.28 sec. The desired speech signal is expected to be enhanced using the aforementioned filters in frequencies 0.1–4.0 kHz, because the linear constrained beamformers may have a low output SNR at low frequencies [18]. We divide this multi-channel signal into 75% overlap frames with 256 samples, and transform them into frequency-domain using a 256-point discrete Fourier transform (DFT). Finally, the output signal of designed filters are transferred into time-domain using the inverse DFT.

The minimum output power beamformer is closely related to the minimum variance beamformer with the distortionless constraint and the perfect signal match [21]. Therefore, the (interference-plus-)noise correlation matrix can be replaced by $\Phi_{\mathbf{y}}(f)$ in the filter designs (10), (12), and (16). We run

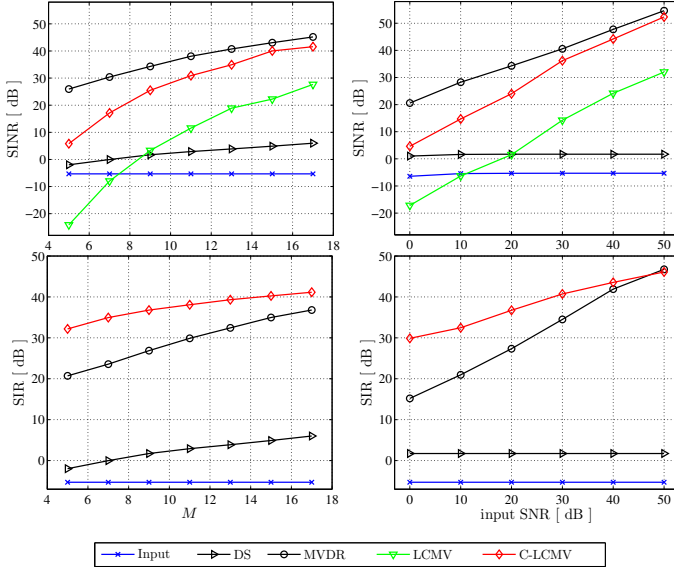


Fig. 2. Output SINR (top row) and output SIR (bottom row) of different beamformers versus number of microphones in 20 dB noise (left column) and versus input SNR level using $M = 9$ (right column).

simulations using different number of microphones and background noise levels. Since two interfering speech signals may have correlation with the desired signal, that is likely, in the C-LCMV beamformer we only null them out and minimize the power of the uncorrelated interfering signal by choosing $N_1 = 3$. Figure 2 shows that the broadband oSINR and oSIR of the C-LCMV beamformer performs in the range between the MVDR and LCMV beamformers.

The expectation is estimated by time averaging, and the correlation matrix of the received signals, at a time instance t , is estimated as

$$\hat{\Phi}_{\mathbf{y},t}(f) = \frac{1}{B} \sum_{b=1}^B \mathbf{y}_{t,b}(f) \mathbf{y}_{t,b}^H(f), \quad (20)$$

where $\mathbf{y}_{t,b}(f)$ is the b th spectral amplitude estimate out of the last B estimates [22]. Moreover, the full rank correlation matrix can be guaranteed by choosing the buffer size as $B \geq M$, and we choose $B = 100$. In practice, the correlation matrix estimate may have error due to the limited number of samples in low iSNRs and the dominant desired signal. Diagonal loading [14] is a solution for this problem, i.e., $\hat{\Phi}_{\mathbf{y}}(f) \leftarrow \hat{\Phi}_{\mathbf{y}}(f) + \gamma \mathbf{I}_M$, that we choose $\gamma = 10^{-4}$. In -5 dB broadband iSINR (20 dB background noise), Figure 3 shows spectrograms of the noisy signal at the first microphone, the output signals of beamformers using $M = 11$ microphones. Although the LCMV beamformer outperforms the MVDR beamformer by removing interferers, the LCMV beamformer distort the speech signal at low frequencies. The

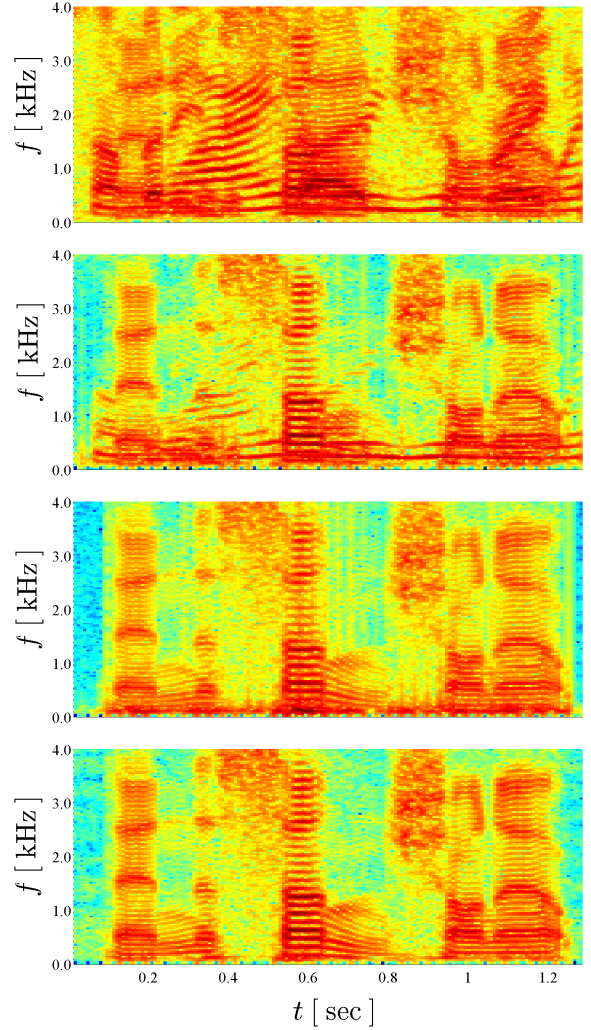


Fig. 3. According to the order of plots from top to down: the spectrograms of the noisy signal at the first microphone, the output signals of the MVDR, LCMV, and C-LCMV beamformers.

experiment results indicate that the C-LCMV beamformer removes interference tracks from the noisy signal without distorting the desired signal at low frequencies.

5. CONCLUSION

The work presented in this paper has focused on signal enhancement in the presence of interference. The LCMV beamformer may have infinite output SIR, but have a lower output SNR than the MVDR beamformer. This problem is increased dramatically using a high number of constraints to remove interferers, especially at low frequencies and closely spaced interference [18]. We have proposed the C-LCMV beamformer being able to control the quality of the signal of interest, a trade-off between noise reduction and interference rejection.

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